Grade 3 Mathematics, Quarter 1, Unit 1.1

Solving Addition and Subtraction Problems Using Strategies Including Place Value and Rounding

Overview

Number of instructional days: 12 (1 day = 45–60 minutes)

Content to be learned

- Use place value understanding to round whole numbers to the nearest 10 or 100.
- Use various strategies and algorithms to add and subtract within 1,000.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Decompose numbers in order to solve addition and subtraction problems.
- Use a number line to conceptualize rounding numbers, and in solving addition and subtraction problems.

Look for and express regularity in repeated reasoning.

- Understand rules for rounding using number lines.
- Evaluate the reasonableness of intermediate results while solving addition and subtraction problems within 1,000.

Essential questions

- How can you round a number to the nearest 10? Explain your thinking.
- How do you round a number to the nearest 100? Explain your thinking.
- What is your strategy for solving a multi-digit addition problem? Explain your thinking.
- How can you explain the steps to solving a multi-digit subtraction problem?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten	**3.NBT**

Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]

[4] A range of algorithms may be used.

3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

There is no prior learning for rounding whole numbers to the nearest 10 or 100.

In second grade, students added and subtracted fluently up to 100 using strategies based on place value, in addition to adding and subtracting up to 1,000 using concrete models or drawings. (2.NBT.5; 2.NBT.7)

Current Learning

Students use place value to understand rounding whole numbers to the nearest 10 or 100. (3.NBT.1) According to PARCC, this is in the developmental stage due to the students not having any prior learning of rounding. This is an additional cluster in third grade.

Students in grade 3 are adding and subtracting within 1,000 using various strategies. (3NBT.2) This is in the reinforcement and drill-and-practice stage. Students should be fluent in adding and subtracting within 1,000 by the end of third grade.

Future Learning

Students in fourth grade will use place value understanding to round multi-digit whole numbers to any place. (4NBT.1) Students in fourth grade will fluently add and subtract multi-digit whole numbers using the standard algorithm. (4.NBT.4)

Additional Findings

According to the book, *Adding It Up: Helping Children Learn Mathematics,* "Estimating the results of a computation is a complex activity that should integrate all strands of mathematical proficiency. Its benefit is realized when students are allowed to draw on other strands to find ways to simplify calculations and compensate for that simplification." (p. 216)

According to *Principles and Standards for School Mathematics,* "Estimation serves as an important companion to computation. It provides a tool for judging the reasonableness of calculator, mental, paper- and pencil computations." (p. 155)

Grade 3 Mathematics, Quarter 1, Unit 1.2

Introducing Multiplication and Its Properties

Overview

Number of instructional days: 12 (for both units 1.2 AND 1.3) (1 day = 45–60 minutes)

Content to be learned

- Interpret products of whole numbers.
- Apply properties of operations as strategies to multiply.
- Identify arithmetic patterns.
- Use properties of operations to explain patterns in arithmetic.

Mathematical practices to be integrated

Attend to precision.

- Explain products as equal numbers in groups.
- Explain the properties of multiplication.

Look for and make use of structure.

- Use the properties of multiplication to make sense of the multiplication process.
- Use arrays and the structure they provide to represent and solve multiplication equations.

Essential questions

- How would you draw an array to represent a multiplication problem?
- Why is it important to use arithmetic patterns in multiplication?
- What would happen to your answer if you switched the order of the factors?
- In what situation would you use multiplication instead of addition? Why?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	**3.OA**

Represent and solve problems involving multiplication and division.

3.OA.1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5×7.*

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.5 Apply properties of operations as strategies to multiply and divide.[2] *Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)*

[2] Students need not use formal terms for these properties.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Common Core Standards for Mathematical Practice

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

Students worked with arrays and wrote addition equations to represent those arrays. (2.OA.4) They have also determined whether or not groups of objects are odd or even and then expressed the groups of even objects as an addition equation with equal addends. (2.OA.3)

Current Learning

Students solve 1-digit multiplication problems. They describe and represent the multiplication problem in context. (3.OA.1) They use properties of multiplication as strategies to solve problems (3.OA.5) Students recognize patterns in multiplication computation and use them to solve problems (3.OA.9) This is a critical area and a major cluster. This learning is in the developmental stage.

Future Learning

Students will apply their knowledge of single digit multiplication to find factors and multiples of numbers 1–100. (4.OA.4) They will begin solving word problems with all four operations that may have remainders. (4.OA.3) Students will also begin multiplying whole numbers and fractions and solving word problems involving multiplication of fractions. (4.NF.4a-c) Multiplication serves as an important foundation for all future mathematics in the elementary and secondary years.

Additional Findings

According to *PARCC Model Content Framework for Mathematics*, “Students must begin work with multiplication and division (3.OA) at or near the very start of the year to allow time for understanding and fluency to develop.” (p. 15)

According to the book *Curriculum Focal Points*, “Students build a foundation for later understanding of functional relationships in context with such statements as, “The number of legs is 4 times the number of chairs.” (p. 15)

According to *Adding It Up: Helping Children Learn Mathematics*, “Finding and using patterns and other thinking strategies greatly simplifies the task of learning multiplication tables.” (p. 191)

"[It is] not clear whether children can be introduced to the division-multiplication relationship very early, thereby learning and practicing quotients at the same time as products, or whether they should learn products first." *A Research Companion to Principles and Standards for School Mathematics* (p. 77)

Grade 3 Mathematics, Quarter 1, Unit 1.3

Exploring the Relationship of Multiplication and Division (Fact Families)

Overview

Number of instructional days: 12 (for both units 1.2 AND 1.3) (1 day = 45–60 minutes)

Content to be learned

- Solve problems involving multiplication and division using whole numbers.
- Multiply and divide within 100.
- Use strategies to solve multiplication and division problems.
- Use fact families to solve for the unknown.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Justify answers to others.
- Define multiplication and division.

Look for and make use of structure.

- Look for patterns and structures in multiplication and division.
- Make connections to prior knowledge of multiplication.

Essential questions

- How can you use fact families to help you solve for an unknown number?
- What is division?
- How do you divide a number? Explain your thinking.
- How can you use a known multiplication fact to solve a division problem?
- What is your strategy for solving the (multiplication/division) problem?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	3.OA

Represent and solve problems involving multiplication and division.

3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations* $8 \times ? = 48$, $5 = \square \div 3$, $6 \times 6 = ?$.

Multiply and divide within 100.

3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In second grade, students worked with arrays and writing addition equations to represent those arrays. (2.OA.4) They also determined whether or not groups of objects are odd or even and then expressed the groups of even objects as an addition equation with equal addends. (2.OA.3)

Current Learning

Students in grade 3 continue to multiply and begin to divide within 100. (3.OA.7) Students are aware of the relationship between multiplication, division, and their properties. Students use their knowledge of fact families and the relationship between multiplication and division to solve for unknown numbers in equations with whole numbers. (3.OA.4) This is a critical area according to PARCC. Understanding division is in the developmental stage and multiplication is in the reinforcement stage in this unit.

Future Learning

In grade 4, students will be expected to find factor pairs for all numbers 1–100. (4.OA.4) Students will also solve multistep word problems using multiplication and division. (4.OA.3) They will use their understanding of division and place value to understand the structure of multi-digit whole numbers. These foundations of multiplication and division will be used through grade 12.

Additional Findings

According to the *PARCC Model Content Framework for Mathematics*, “Students must begin work with multiplication and division (3.OA) at or near the very start of the year to allow time for understanding and fluency to develop.” (p. 15)

A Research Companion to Principles and Standards for School Mathematics states, “Not clear are whether children can be introduced to the division-multiplication relationship very early, thereby learning and practicing quotients at the same time as products, or whether they should learn products first.” (p. 77)

Grade 3 Mathematics, Quarter 1, Unit 1.4

Creating Picture and Bar Graphs to Represent Data

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Represent a data set with several categories by drawing a scaled picture graph.
- Represent a data set with several categories by drawing a scaled bar graph.

Mathematical practices to be integrated

Model with mathematics.

- Identify data on a picture graph or bar graph.
- Use data to make a representation.
- Analyze the relationships of the data to draw conclusions.

Use appropriate tools strategically.

- Select appropriate tools to solve problems.
- Use bar graphs and picture graphs to represent data.

Essential questions

- What does the data represent?
- How would you represent the data on a bar graph?
- How would you represent the data on a picture graph?
- Why do you think that information is put into graph form?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	3.MD

Represent and interpret data.

3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs.-*For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

First grade students practiced categorizing data into three categories and compared those categories. (1.MD.4) In second grade, students began drawing single unit scale pictures and bar graphs to represent data into four categories. (2.MD.10)

Current Learning

Students represent data in a given set of data on bar graphs and picture graphs. These graphs have several categories (6 or fewer) and each picture or block on the graphs may represent multiple objects. (3.MD.3) This is a supporting cluster and is in the developmental stage.

Later, in third grade, students use their knowledge of bar graphs and picture graphs to solve one- and two-step problems. They also generate their own data to create picture and bar graphs. (3.MD.4)

Future Learning

In grade 4, students will represent and interpret data using a line plot. In grade 5, they will use operations on fractions to solve problems involving information presented on a line plot. Students' work with categorical data in early grades will develop into later work with bivariate categorical data and two-way tables in eighth grade. (8.SP.4)

Additional Findings

"Understanding data representation and analysis involves many complex issues, from sorting through what different numbers on a graph mean, to choosing appropriate measures to summarize and compare groups, to identifying relationships between variables. Through multiple experiences with a variety of data sets, students begin to develop the tools and concepts they need to use data themselves and to interpret the data they will encounter throughout life" *A Research Companion to Principles and Standards for School Mathematics.* (p. 213)

Curriculum Focal Points states, "Addition, subtraction, multiplication, and division of whole numbers come into play as students construct and analyze frequency tables, bar graphs, picture graphs, and line plots and use them to solve problems." (p. 15)

Grade 3 Mathematics, Quarter 1, Unit 1.5

Telling Time and Solving Problems With Time Intervals

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Tell and write time to the nearest minute.
- Measure time intervals in minutes.
- Solve word problems involving addition of time intervals in minutes.
- Solve words problems involving subtraction of time intervals in minutes.
- Represent problems on a number line diagram.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Use concrete objects (analog clocks and number lines) to help them make sense of problems and solve them.
- Draw pictures to represent the problem and explain the relationship to the problem.

Model with mathematics.

- Use time to solve problems in real-world situations.
- Write an equation to describe the problem situation.

Essential questions

- What information does the minute/hour hand give you?
- How can you explain how to tell time?
- How do you know when the time has been written correctly?
- How can you find how much time has passed?
- How would you use a number line to show how much time has elapsed?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	**3.MD**

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

3.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

Students in first grade learned to tell time to the nearest hour and half-hour. (1.MD.3) Students in second grade learned to tell time to the nearest 5 minutes on analog and digital clocks and learned to distinguish between AM and PM. (2.MD.7) First and second grade students have solved addition and subtraction problems and word problems. (1.OA.1, 1.OA.2, 2.OA.1, 2.OA.2)

Current Learning

Third graders are expected to tell and write time to the nearest minute and measure time intervals in minutes. Students solve word problems in addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram (3.MD.1).

The instructional level is developmental and reinforced during future learning. Students are required to tell time to the nearest minute instead of the nearest 5 minutes. This unit extends students prior learning of five-minute intervals to one-minute intervals. This is not a critical area of instruction for third grade, but is a major cluster according to the PARCC Frameworks.

Future Learning

Students in grade 4 will use the four operations to solve word problems in intervals of time and other measurements given in a larger unit in terms of a smaller unit. (4.MD.2) Students will need this skill in order to be successful in their future mathematical endeavors and everyday life.

Additional Findings

According to *Principles and Standards for School Mathematics*, "Emphasis should be on developing concepts of time and the ways it is measured. (p. 104) The measurement process is identical, in principle, for measuring any attribute: choose a unit to the object, and report the number of units. The number of units can be determined by *iterating* the unit and counting the iterations or by using a measurement tool," (p. 105).

Grade 3 Mathematics, Quarter 2, Unit 2.1

Developing Fluency Up to 1,000 in Addition and Subtraction

Overview

Number of instructional days: 7 (1 day = 45–60 minutes)

Content to be learned

- Fluently add and subtract by decomposing numbers based on place value.
- Use strategies based on the relationship of addition and subtraction to fluently add and subtract within 1,000.
- Use properties of operations to fluently add and subtract within 1,000.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Activate prior knowledge.
- Analyze the problem.
- Use representation to help solve the problem.

Attend to precision.

- Use appropriate vocabulary.
- Attend to details.

Essential questions

- What are the steps involved in solving an addition problem using place value or decomposing numbers?
- How could you use the inverse operation to solve an addition or subtraction problem?
- How could you use the properties of operations to solve an addition or subtraction problem?
- How can you use what you know about addition to help you solve subtraction problems?
- How can you use what you know about subtraction to help you solve addition problems?
- How does your understanding of place value help you solve addition and subtraction problems?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten	3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]

[4] A range of algorithms may be used.

3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In second grade, students added and subtracted fluently up to 100 using strategies based on place value; they also added and subtracted up to 1,000 using concrete models or drawings. (2.NBT.5; 2.NBT.7)

Current Learning

Third-grade students add and subtract within 1,000 using various strategies (3NBT.2). This is in the reinforcement and drill and practice stage. Students should be fluent in adding and subtracting within 1,000 by the end of third grade.

Future Learning

Students in fourth grade will fluently add and subtract multi-digit whole numbers using the standard algorithm. (4.NBT.4)

Additional Findings

According to *Principles and Standards for School Mathematics*, "As students move from third to fifth grade, they should consolidate and practice a small number of computational algorithms for addition, subtraction, multiplication, and division that they understand well and can use routinely." (p. 155)

Also according to *Principles and Standards for School Mathematics,* "Regardless of the particular algorithm used, students should be able to explain their method and understand that many methods exist." (p.155)

"Research suggests that students are capable of listening to their peers and to their teacher and of making sense of an algorithm if it is explained and if the students have diagrams or concrete materials that support their understanding of the quantities involved." (*Adding It Up*, p. 197)

Grade 3 Mathematics, Quarter 2, Unit 2.2

Understanding and Using Division and Its Properties and Relationship to Multiplication to Solve Problems

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Interpret whole-number quotients as partitioned, equal shares of a group.
- Apply properties of operations as strategies to multiply and divide.
- Understand division as an unknown-factor problem.

Mathematical practices to be integrated

Look for and make use of structure.

- Use structure to understand and make sense of properties of multiplication and division.

Look for and express regularity in repeated reasoning.

- Notice repeated calculations.
- Look for general methods and shortcuts.

Essential questions

- How can knowing a fact family help you create or solve multiplication and division problems?
- How can you use your knowledge of multiplication facts to solve for an unknown factor in a division problem?
- How can you find the unknown factor when given an array with the total units and the dimensions of one side?
- What is a quotient?
- How can you use multiplication to solve a division problem?
- What is your strategy for solving this division problem?
- How can you model this division problem using manipulatives, objects, or pictures?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	3.OA

Represent and solve problems involving multiplication and division.

3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.5 Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

[2] Students need not use formal terms for these properties.

3.OA.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

Students worked with arrays and writing addition equations to represent arrays with dimensions of up to five rows and columns (2.OA.4). Students also determined whether or not groups of objects were odd or even and expressed the groups of even objects as an addition equation with equal addends up to 20 (2.OA.3).

Current Learning

Students are beginning to interpret whole-number quotients as partitioned equal shares of a group. They are also beginning to see the relationship between multiplication and division and their properties. They use this knowledge to solve unknown-factor division problems (3.0A2, 3.AO.5, 3.OA.6). This is a critical area and a major cluster for third graders. Developing a conceptual understanding of division should be the primary focus of this unit. The level of instruction in this unit is developmental for division and reinforcement for multiplication.

Future Learning

In fourth grade, students will solve word problems involving multiplication and division (4.OA.2) and understand how the operations compare (example: 3 x ? = 18 or 18 ÷ ? = 3). Students will also find whole-number quotients and remainders using their knowledge of the relationship between multiplication and division. (4.NBT.6) Students will need a good foundation in multiplication and division to move to multiplication reasoning in the next grades and then to proportional reasoning in the upper grades.

Additional Findings

According to *PARCC Model Content Framework for Mathematics*, "Students must begin work with multiplication and division (3.OA) at or near the very start of the year to allow time for understanding and fluency to develop." (p.15)

"Not clear is whether children can be introduced to the division–multiplication relationship very early, thereby learning and practicing quotients at the same time as products, or whether they should learn products first." *A Research Companion to Principles and Standards for School Mathematics* (p.77)

Grade 3 Mathematics, Quarter 2, Unit 2.3

Applying Multiplication and Division Within 100 to Solve Problems

Overview

Number of instructional days: 9 (1 day = 45–60 minutes)

Content to be learned

- Represent and solve multiplication and division word problems involving equal groups, arrays, and measurement quantities.
- Develop strategies to multiply and divide within 100.
- Represent and interpret data using scaled bar and picture graphs with several categories to solve one- and two-step problems.
- Represent and solve word problems involving multiplication and division using arrays, drawings, and equations.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Use concrete representations to explain and defend a solution pathway.
- Justify conclusions and communicate them to others.
- Respond to arguments of others.

Use appropriate tools strategically.

- Use tools (such as arrays, drawings, bar graphs, and picture graphs) appropriately to assist in solving one- and two-step problems involving multiplication, division, addition, and subtraction.
- Decide which tools are the most appropriate for the problems they are solving.

Essential questions

- What strategies can be used to solve for unknowns in a two-step word problem?
- How can drawing a graph help you understand a data set with several different categories?
- What model could you use to construct and represent a problem?
- How does your model, drawing, or array represent this problem?
- How can you use the information in the graph to solve this problem?
- How do you decide what scale to use for your bar or picture graph?
- When would you choose to use a scaled graph instead of a single scaled graph?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	3.OA

Represent and solve problems involving multiplication and division.

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

[1] See Glossary, Table 2.

Multiply and divide within 100.

3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Measurement and Data	3.MD

Represent and interpret data.

3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

Students worked with arrays with up to five rows and columns and writing addition equations to represent those arrays (2.OA.4). Students also determined whether or not groups of objects are odd or even and then expressed the groups of even objects as an addition equation with equal addends up to 20 (2.OA.3). Students drew single-unit scale pictographs to represent data from bar graphs and solved simple problems (2.MD.10).

Current Learning

Students use their knowledge of multiplication and division to solve word problems with solutions within 100. Students also learn to fluently multiply and divide within 100. They will know from memory all of their multiplication and division facts by the end of the year (3.OA.3, 3.OA7). Students are now able to use scaled picture and bar graphs to solve one-and two-step word problems with their data (3.MD.3). Since students have practiced multiplication, division, and graphs but have not really applied their knowledge to solve and one- and two-step word problems, most of this is in the developmental stage. (See Table 2, Common Core State Standards, p. 89, for sample questions to use in the classroom; do not overlook the area example problems.) Students should be fluent to multiply and divide within 100 from memory by the end of third grade.

Future Learning

In fourth grade students will solve word problems involving multiplication and division (4.OA.2) and understand how the operations compare (example: 3 x ? = 18 or 18 ÷ ? = 3). Students will also find whole-number quotients and remainders using their knowledge of the relationship between multiplication and division. (4.NBT.6) Students will need a good foundation in multiplication and division to move to multiplication reasoning in the next grades and then to proportional reasoning in the upper grades.

Additional Findings

According to *PARCC Model Content Framework for Mathematics*, “the most important development in data representation for categorical data is that students now draw picture graphs in which each picture represents more than one object, and they draw bar graphs in which the height of a given bar in tick marks must be multiplied by the scale factor in order to yield the number of objects in the given category. Students can draw a scaled picture graph or a scaled bar graph to represent a data set with several categories (3.MD.3). They can solve one- and two-step “how many more?” and “how many less?” problems using information presented in scaled bar graphs” (3.OA.3, 3.OA.8) (p.7).

According to *Principles and Standards for School Mathematics*, “As students learn to describe the similarities and differences between data sets, they will have an opportunity to develop clear descriptions of the data sets, they will have an opportunity to develop clear descriptions of the data and to formulate conclusions and arguments based on the data” (p. 177).

Grade 3 Mathematics, Quarter 2, Unit 2.4

Measuring Area Using Unit Squares

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Recognize a unit square.
- Recognize area as a type of measurement.
- Measure the area of a plane figure using unit squares with no gaps or overlaps.
- Count unit squares to determine the area of a rectangle.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Students form arrays in equal rows and columns in rectangular figures to find area

Use appropriate tools strategically.

- Students are using tiles to measure area of plane figures.

Look for and express regularity in repeated reasoning.

- The same strategy will be used each time to find area with unit squares.
- Evaluate reasonableness of answers.

Essential questions

- How would you use unit squares to measure the area of a rectangle?
- What does area tell you about the attribute of the figure?
- Why can your tiles not have gaps or overlaps when measuring area?
- What is a square unit of measure? How do square inches differ from inches?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	3.MD

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.

b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

Common Core State Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

Students partitioned rectangles into rows and columns (2.G.2) in second grade and arrange objects into rectangular arrays up to 5 x 5 (2.OA.4).

Current Learning

In this unit, students learn area is an attribute of plane figures. They learn how to measure area by counting unit squares. Students find the area of a rectangle and relate the area to multiplication and addition (3.MD.7). This is a critical area of learning for third grade and is in the developmental stage.

Future Learning

In fourth grade, students will use area formulas to solve real-world math problems (4.MD.3). Students will use their knowledge of counting unit squares to find area and volume (5.MD.3).

Additional Findings

According to *PARCC Progressions K–5, Geometric Measurement*, students need to learn to conceptualize area as the amount of two-dimensional space in a bounded region and to measure it by choosing a unit of area, often a square (p. 16).

Grade 3 Mathematics, Quarter 2, Unit 2.5

Solving Area Problems Using Tools and Strategies

Overview

Number of instructional days: 7 (1 day = 45–60 minutes)

Content to be learned

- Find the area of a rectangle through tiling and explain how the area equals the product of the side lengths.
- Use multiplication to find the area of rectangles in real-world problems.
- Use tiling as a tool to prove that area can be figured by multiplying side lengths.
- Decompose a larger rectangle into smaller rectangles and find the area of the smaller rectangles, then add the smaller areas to arrive at the area of the larger rectangle.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Use math tiles and drawings to find the area of rectangles.

Look for and make use of structure.

- Recognize that a larger rectangle can be broken up into smaller rectangles to find the area.
- Look closely to discern a pattern, conceptualizing the connection between area and multiplying side lengths of rectangles.

Essential questions

- How would we use your knowledge of finding the area of a rectangle outside of school?
- How do you decompose a large rectangle into smaller rectangles to find the area?
- How does finding the area of smaller rectangles help to figure the area of a larger rectangle?
- Why does multiplying side lengths work to find the total number of tiles in a rectangle?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	**3.MD**

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.7 Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Common Core Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

Students partitioned rectangles into rows and columns (2.G.2) in second grade and arranged objects into rectangular arrays up to 5 x 5 (2.OA.4).

Current Learning

Students in third grade have been working with square unit tiles to learn how to measure the area of plane figures. They tile a rectangle with no overlaps or gaps and correctly count the unit squares to find the area.

In this unit, students see the relationship between area and the multiplication of sides. They use their knowledge to solve real-world area problems. They are also learning to decompose larger rectangles into smaller rectangles to find area.

Future Learning

Students will apply area formulas to solve real-world mathematical formulas in fourth grade (4.MD.3). In fifth grade, students will be expected to apply their knowledge of area formulas to compute the volume of 3-D figures (5.MD.5a,c). In sixth grade, students will be expected to solve problems involving area, surface, and volume. They will have to find the measures for many shapes, including triangles, polygons, and quadrilaterals.

Additional Findings

According to *Principles and Standards for School Mathematics,* “Students in grade 3–5 should develop strategies for determining surface area and volume on the basis of concrete experiences. They should measure various rectangular solids using objects (such as tiles and cubes), organize the information, look for patterns, and then make generalizations” (p.175).

According to *PARCC K–5, Geometric Measurement*, "Third graders focus on learning area, Students learn formulas to compute area, with those formulas based on, and summarizing, a firm conceptual foundation about what area is. Students need to learn to conceptualize area as the amount of two- dimensional space in a bounded region and to measure it by choosing a unit of area, often a square. (p. 16)."

Grade 3 Mathematics, Quarter 3, Unit 3.1

Understanding Fractions as Numbers

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Understand that a fraction is a part of a whole when that whole is divided into equal parts.
- Partition many different shapes with the understanding that each partition is a fraction of the whole.
- Label each equal part of the whole using fraction notation. (1/2, 1/3, ¼, 1/6, 1/8)
- Introduce numerator and denominator vocabulary.
- Understand that a fraction a/b is composed of "*a*" quantity of unit fractions the size of *1/b*.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Use manipulatives to make coherent representations of fractions.
- Make sense of quantities and their relationships.

Model with mathematics.

- Use manipulatives or drawings to model representations of fractions.
- Use fraction notation to describe a situation.

Essential questions

- How can you use this (manipulative) to show halves (thirds, fourths, etc.)?
- How do you know this partition shows halves (thirds, fourths, etc.)?
- How would you partition this shape into halves (thirds, fourths, etc.)?
- If a whole is partitioned into (four, five, etc.) equal parts, how would you know how to name that fraction?
- How do you know this piece is the size 1/2 (1/3, ¼, 1/6, 1/8)?
- How many ¼'s are needed to make a whole? How do you know?
- How many ¼'s are needed to represent 3/4s? Explain using manipulatives.
- What does the denominator represent? What does the numerator represent?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	**3.G**

Reason with shapes and their attributes.

3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Evidence Statement: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Number and Operations—Fractions[5]	**3.NF**

[5] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

Develop understanding of fractions as numbers.

3.NF.1 Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

Evidence Statement: Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In second grade, students partitioned circles and rectangles into two, three, or four equal shares; described the shares using the words *halves*, *thirds*, *half-of*, *third of*, etc.; described the whole as two halves, three thirds, four fourths; and recognized equal shares of identical wholes (2.G.3).

Current Learning

Students in third grade understand a fraction as the quantity formed by one part when a whole is partitioned into *b* equal parts. Students understand a fraction *a/b* as the quantity formed by parts of size 1/b (3.NF.1). This is a major critical area and major cluster for third grade. Third-grade students partition shapes into parts with equal areas and express the area of each part as a unit fraction of the whole (3.G.2). Students are also introduced to formal fraction notation. They develop understanding of the relationship of numerators and denominators to the size of the fraction and the relationship to the whole. Students are working at a developmental level of instruction.

Future Learning

Students in fourth grade will understand that a fraction *a/b* with *a*>1 is a sum of fractions 1/b. They will understand addition and subtraction of fractions as joining and separating parts referring to the same whole. Students will decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each denominator with an equation (4.NF.3ab).

Additional Findings

According to *Principles and Standards for School Mathematics*, “During grades 3–5, students should build their understanding of fractions as parts of a whole and as division. They will need to see and explore a variety of models of fractions, focusing primarily on familiar fractions such as halves, thirds, fourths, fifths, sixths, eighths, and tenths” (p.150).

According to *PARCC Progressions K-5, Numbers and Operations in Base Ten,* the goal for students is to see unit fractions as the basic building blocks of fractions, in the same sense that the number 1 is the basic building block of the whole numbers.

According to *PARCC Progressions 3–5 Number and Operations: Fractions*, “Grade 3 students start with unit fractions (fractions with numerator 1), which are formed by partitioning a whole into equal parts and taking one part, e.g., if a whole is partitioned into 3 equal parts then each part is ¼ of the whole, and 4 copies of that part make the whole. Next, students build fractions from unit fractions, seeing the numerator 3 of ¾ as saying that ¾ is the quantity you get by putting 3 of the ¼’s together. They read any fraction this way, and in particular there is no need to introduce “proper fractions” and “improper fractions” initially; 5/3 is the quantity you get by combining 5 parts together when the whole is divided into 3 equal parts (p. 2).

Grade 3 Mathematics, Quarter 3, Unit 3.2

Generating Linear Measurement Data Using Fractions

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Partition an interval on a number line in 2, 3, 4, 6, or 8 equal parts.
- Recognize that each partition is represented by a fractions 1/*b* with *b* representing the number of partitions.
- Represent fractions with a numerator other than 1 on a number line with *b* still being the number of partitions in the interval.
- Generate measurement data with rulers in half and quarter inches.
- Represent data on a line plot with a scale marked off in whole numbers, halves, and quarters.

Mathematical practices to be integrated

Model with mathematics.

- Represent measurements using fractions.
- Use manipulatives or drawings to model representation of fractions.

Use appropriate tools strategically.

- Students are able to use line plots, rulers, and number lines to deepen their understanding of fractions.

Attend to precision.

- Use appropriate vocabulary to describe the fractions.
- Calculate measurements accurately and efficiently.

Essential questions

- How would you partition this line into equal parts?
- How can you use this number line to show halves, quarters, etc.?
- How would you represent data on a line plot using fractions?
- When collecting data, why would you decide to measure to the half or quarter inch, not just to the whole inch?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions[5]	**3.NF**

[5] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

Develop understanding of fractions as numbers.

3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.

b. Represent a fraction *a/b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.

Measurement and Data	**3.MD**

Represent and interpret data.

3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

Common Core State Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

Students worked with number lines in first grade and with rulers in second grade. They represent equally spaced whole numbers on a number line (2.MD.6). Students generated measurement data in whole number units and represented that data on a line plot with a scale that is marked off in whole number-units in second grade (2.MD.9).

Current Learning

Students are knowledgeable of fractions and understand that a fraction is a part of a whole that is divided into equal parts. They partition many different shapes with the understanding that each partition is a fraction of a whole. Students understand that a fraction a/b is composed of "a" quantity of unit fractions the size of $1/b$ (3.NF.1). Students are representing fractions on a number line and are developing a deeper understanding of fractions as part of a whole (3.NF.2). They are now generating measurement in fractional units and representing that data on a line plot with a scale marked in fraction units (3.MD.4). Students continue to work with fractions in third grade by comparing, explaining equivalence, and using visual models (3.NF.3). This is a major cluster and a critical area for third grade. This is in the developmental stage due to students' limited knowledge of fractions.

Future Learning

In fourth grade, students will extend their understanding of fraction equivalence and ordering. They will build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers, will understand decimal notations for fractions with denominators of 10 or 100, and will compare decimal fractions (4.NF).

Additional Findings

According to *Principles and Standards for School Mathematics*, "During grades 3–5, students should build their understanding of fractions as parts of a whole and as division. They will need to see and explore a variety of models of fractions, focusing primarily on familiar fractions such as halves, thirds, fourths, fifths, sixths, eighths, and tenths" (p. 150).

According to *PARCC Progressions K–5, Numbers and Operations in Base Ten,* The goal for students to see unit fractions as the basic building blocks of fractions, in the same sense that the number 1 is the basic building block of the whole numbers.

Grade 3 Mathematics, Quarter 3, Unit 3.3

Comparing Fractions with Models and Reasoning

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Understand that different fractions that are the same size or have the same point on a number line are equivalent.
- Recognize fractions that are equivalent to whole numbers.
- Write fractions that represent whole numbers, and write a whole number as a fraction.
- Compare two fractions having the same whole, and use symbols greater than, less than, and equal to (<, >, =) to record the comparison.
- Use concrete models or manipulatives to reason about the sizes of fractions.
- Reason about the size of fractions with like numerators or denominators.
- Generate simple equivalent fractions, and explain the equivalence using visual models.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Decompose a whole number into equal parts resulting in a fraction.
- Use greater than, less than, and equal symbols to compare fractions.

Construct viable arguments and critique the reasoning of others.

- Use logic to analyze problems in order to find parts of a whole.
- Use a number line to represent that fractions are numbers.

Look and make use of structure.

- Connect to prior knowledge.
- See things as a single object or as a whole composed of several parts.

Essential questions

- How would you demonstrate that two fractions are equivalent?
- How does the number in the denominator affect the size of the fraction?
- What does a 1 in the denominator represent?
- How would you compare two fractions with the same numerator?
- How would you compare two fractions with the same denominator?
- How would you write a whole number as a fraction?
- How can you use a visual model to prove fractions are equivalent or not?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions[5]	**3.NF**

[5] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

Develop understanding of fractions as numbers.

3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Common Core State Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Clarifying the Standards

Prior Learning

Second graders worked on partitioning circles and rectangles into two, three, or four equal parts. They used words such as halves, thirds, half of, a third of, etc. (2.G.3)

Current Learning

Students explain the equivalence of fractions and compare fractions by reasoning about their size. They understand that two fractions are equivalent in size by visual models and using a number line. Students will recognize equivalent fractions (1/2 = 2/4 and 4/6 = 2/3,) and be able to compare greater than, less than, and equal fractions. This is a major cluster as well as a critical area for third grade. This will be a reinforcement as well as drill-and-practice for third grade.

Future Learning

In fourth grade, students will explain why a fraction a/b is equivalent to a fraction $(n/a)(n \text{ x } b)$ and be able to compare two fractions with different numerators and denominators by creating common numerators and denominators. They will adding and subtracting fractions with like denominators, and will be solving word problems involving adding, subtracting, and multiplying fractions by a whole number They will also record decimal fractions using decimal notation. (4.NF.1, 2, 3a–d, 4a–c, 5, 6, 7).

Additional Findings

According to *PARCC Progressions K–5, Numbers and Operations—Fractions*, "students need to see unit fractions as the basic building blocks of fractions, in the same sense that the number 1 is the basic building block of whole numbers" (p. 3).

According to *Principles and Standards for School Mathematics*, "By using an area model in which part of a region is shaded, students can see how fractions are related to a unit whole, comparing fractional parts of a whole, and find equivalent fractions" (p. 150).

Grade 3 Mathematics, Quarter 3, Unit 3.4

Solving Problems Using Tools and Strategies

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Solve two-step word problems using the four operations (addition, subtraction, multiplication, and division).
- Represent story problems with equations, using a letter to stand for the unknown variable.
- Use estimation strategies (including rounding) and mental computation to assess the reasonableness of answers.
- Use place value strategies and multiplication operations to multiply one digit of 10 within a range of 10–90.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Analyze what is given in the word problem in order to explain the meaning of the problem.
- Organize and represent the word problem with equations.

Reason abstractly and quantitatively.

- Create a logical representation of the problem.
- Solve word problems and computations by understanding quantities and being flexible in the use of operations and their properties.

Construct viable arguments and critique the reasoning of others.

- Justify conclusions with mathematical reasoning.
- Use logic to analyze problems in order to decide which operations to use.

Essential questions

- How did you decide which operations to use in the problem?
- How did you decide what the problem was asking you to find?
- How do you represent the word problem using an equation?
- How do you know if your solution is reasonable?
- What pattern do you see when multiplying one-digit numbers by 10?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	**3.OA**

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3]

[3] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Number and Operations in Base Ten	**3.NBT**

Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]

[4] A range of algorithms may be used.

3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Clarifying the Standards

Prior Learning

In first grade, students began to understand the structure of base ten number systems and that multiples of 10 represent the number of tens in that number. First graders also learned to subtract multiples of 10 from multiples of 10 using place value strategies (1.NBT.5).

In second grade, students mentally added and subtracted 10s from numbers using their place value knowledge (2.BT.8). Second graders also used addition and subtraction to solve one- and two-step problems (2.OA.1). These problems have involved unknowns in all positions. In second grade, students were able to explain when addition and subtraction strategies work using their knowledge of place value and the properties of operations (2.NBT.9).

Current Learning

In third grade, students work with whole numbers to answer two-step word problems using the four operations. They use a letter to represent the unknown variable. The students use mental computation and estimation strategies, including rounding, to find answers. They also work out multiplication problems by

solving one-digit whole numbers with multiples of 10. This work with story problems is in the reinforcement stage, but using two-step problems is at the developmental stage (3.0A.8, 3.NBT.3).

Future Learning

In fourth grade, students will solve multistep word problems involving all four operations with answers that may have remainders (4.OA.3). Students will continue using place value and order of operations to divide four-digit numbers by one-digit numbers with remainders. They will also use the relationship between multiplication and division to solve division problems. (4.NBT.6) Students will be able to distinguish between multiplicative comparison and additive comparison in problem solving. (4.OA.2)

Additional Findings

According to *Principles and Standards for School Mathematics,* "Good problems and problem-solving tasks…generally serve multiple purposes, such as challenging students to develop and apply strategies, introducing them to new concepts, and providing a context for using skills" (p. 183).

According to *PARCC*, "Use of two-step problems involving easy or middle difficulty adding and subtracting within 1,000, or one such adding or subtracting problem with one step of multiplication or division, can help maintain fluency with addition and subtraction while giving the needed time to the major grade 3 multiplication and division standards."

Grade 3 Mathematics, Quarter 3, Unit 3.5

Solving Measurement Problems Involving Time, Volume, Perimeter, and Mass

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Measure and estimate liquid volume and mass of objects using grams, kilograms, and liters.
- Solve one-step word problems involving masses or liquid volumes using all four operations.
- Solve real-world problems involving perimeter of polygons.
- Show different rectangles that share equal perimeters or areas.
- Find perimeter of polygons with known or missing side lengths.
- Solve perimeter problems involving rectangles having the same areas but different perimeters.
- Solve perimeter problems involving rectangles having the same perimeters but different areas.
- Distinguish between linear and area measures.

Mathematical practices to be integrated

Model with mathematics.

- Represent the quantities in standard units of grams, kilograms, and liters.
- Make sense of linear and area measures through the use of models and drawings.
- Apply mathematics they know to solve problems arising in everyday life.

Look for and express regularity in repeated reasoning.

- Evaluate the reasonableness of their intermediate results.
- Recognize that attributes of polygons can give clues to finding perimeter when a side length is missing.
- Look for shortcuts.

Essential questions

- How does your estimated measurement compare to the actual measurement?
- How do you know your measurement is accurate?
- How do you choose a unit of measurement to use when measuring?
- How would you show that the areas or perimeters of two different rectangles are equal?
- How could you use the information you have to find the length of the missing side of this shape?
- How can you use a mode, drawing, or equation to solve a story problem involving mass or volume?
- What is the difference between linear measure (perimeter) and area measure?
- What is your strategy for solving this problem?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	3.MD

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[6] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[7]

[6] Excludes compound units such as cm^3 and finding the geometric volume of a container.

[7] Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2).

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In second grade, students solved word problems involving lengths using addition and subtraction within 100 and by using drawings (2.MD.5). Students had no prior learning on measuring mass and volume (3.MD.2).

Current Learning

Students are learning to measure liquid volume and masses of objects. They are also solving one-step word problems involving mass and volume by using drawings to represent the problem. They solve problems involving perimeters and areas of polygons. This is in the developmental stage. It is also a major cluster and a critical area.

Future Learning

Students will apply area and perimeter formulas to solve real-world mathematical formulas in fourth grade (4.MD.3). They will solve word problems involving volume and mass (4.MD.2).

Additional Findings

According to *Principles and Standards for School Mathematics,* “Students in grade 3–5 should develop strategies for determining surface area and volume on the basis of concrete experiences. These concrete experiences are essential in helping students understand the relationship between the measurement of an object and the succinct formula that produces the measurement” (p. 175).

According to *PARCC K–5, Geometric Measurement*, Third graders can learn to measure liquid volume and to solve problems requiring the use of four arithmetic operations, when liquid volumes are given in the same units throughout each problem. Because liquid measurement can be represented with one-dimensional scales, problems may be presented with drawings or diagrams, such as measurements on a beaker with a measurement scale in milliliters.

Grade 3 Mathematics, Quarter 3, Unit 3.6
Understanding 2-D Shapes and Their Attributes

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Categorize quadrilaterals based on their attributes.
- Understand that some quadrilaterals may belong to many different categories because of shared attributes.
- Draw quadrilaterals that do not belong in any subcategory.

Mathematical practices to be integrated

Attend to precision.

- Draw examples of quadrilaterals.
- Recognize rhombi, rectangles, and squares as examples of quadrilaterals.
- Identify shapes and their attributes.

Look for and make use of structure.

- Recognize the similarities in shapes, focusing on their attributes.
- Use prior knowledge to define shapes according to their attributes.

Essential questions

- When comparing two shapes, how do you use their attributes to describe their differences?
- When comparing two shapes, how do you use their attributes to find what they have in common?
- How would you draw a quadrilateral that does not belong in any other group?
- What are some categories this shape belongs to? What attributes determine the category?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	**3.G**

Reason with shapes and their attributes.

3.G.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Common Core Standards for Mathematical Practice

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In kindergarten, students compared two- and three-dimensional shapes of different sizes and attributes (K.G.4). In first grade, they distinguished between defining and non-defining attributes (1.G.1). In second grade, students recognized shapes by their attributes. They identified quadrilaterals and identified the number of angles and faces (2.G.1).

Current Learning

Students are working with shapes in different categories that share attributes. They draw examples of quadrilaterals, recognizing that rhombi, rectangles, and squares are examples of quadrilaterals. This is a critical area and is in the reinforcement stage.

Future Learning

In fourth grade, students will look at two-dimensional figures and will draw points, lines, line segments, rays, angles, and perpendicular and parallel lines (4.G.1).

Additional Findings

According to *Principles and Standards for School Mathematics,* "In grades 3–5, teachers should emphasize the development of mathematical arguments. As students' ideas about shapes evolve, they should formulate conjectures about geometric properties and relationships. Using drawings, concrete materials, and geometry software to develop and test their ideas, they can articulate clear mathematical arguments about why geometric relationships are true (p. 166).

According to *Adding It Up*, "Children enter school with much informal knowledge of geometry that can be developed throughout the grades" (p. 288).